

Yale University

## EliScholar – A Digital Platform for Scholarly Publishing at Yale

---

Cowles Foundation Discussion Papers

Cowles Foundation

---

12-1-1999

### Coordination Risk and the Price of Debt

Stephen Morris

Hyun Song Shin

Follow this and additional works at: <https://elischolar.library.yale.edu/cowles-discussion-paper-series>



Part of the [Economics Commons](#)

---

#### Recommended Citation

Morris, Stephen and Shin, Hyun Song, "Coordination Risk and the Price of Debt" (1999). *Cowles Foundation Discussion Papers*. 1490.

<https://elischolar.library.yale.edu/cowles-discussion-paper-series/1490>

This Discussion Paper is brought to you for free and open access by the Cowles Foundation at EliScholar – A Digital Platform for Scholarly Publishing at Yale. It has been accepted for inclusion in Cowles Foundation Discussion Papers by an authorized administrator of EliScholar – A Digital Platform for Scholarly Publishing at Yale. For more information, please contact [elischolar@yale.edu](mailto:elischolar@yale.edu).

**Coordination Risk and the Price of Debt**

**By**

**Stephen Morris and Hyun Song Shin**

**December 1999**

**COWLES FOUNDATION DISCUSSION PAPER NO. 1241**



**COWLES FOUNDATION FOR RESEARCH IN ECONOMICS**

**YALE UNIVERSITY**

**Box 208281**

**New Haven, Connecticut 06520-8281**

**<http://cowles.econ.yale.edu/>**

# Coordination Risk and the Price of Debt\*

Stephen Morris  
Cowles Foundation,  
Yale University,  
P.O.Box 208281,  
New Haven CT 06520, U. S. A.

`stephen.morris@yale.edu`

Hyun Song Shin  
Nuffield College,  
Oxford University,  
Oxford, OX1 1NF,  
U. K.

`hyun.shin@nuf.ox.ac.uk`

Revised, November 1999

## Abstract

Creditors of a distressed borrower face a coordination problem. Even if the fundamentals are sound, fear of premature foreclosure by others may lead to pre-emptive action, undermining the project. Recognition of this problem lies behind corporate bankruptcy provisions across the world, and it has been identified as a culprit in international financial crises, but has received scant attention from the literature on debt pricing. The apparent multiplicity of equilibria is a barrier to development of this issue in asset pricing, but this multiplicity is only apparent. Without common knowledge of fundamentals, the incidence of failure is uniquely determined provided that private information is precise enough. This affords a way to price the coordination failure. There are two further conclusions. First, coordination is more difficult to sustain when fundamentals deteriorate. Thus, when fundamentals deteriorate, the onset of crisis can be very swift. Second, “transparency” - in the sense of greater provision of information to the market - does not generally mitigate the coordination problem. Transparency is not a panacea.

---

\*We are indebted to many colleagues for their encouragement and advice. Jean-Charles Rochet, Jürgen Eichberger, Javier Suarez and Stefan de Wachter acted as discussants of this paper at conferences, and have left their mark on the current version. We also learned much from Patrick Bolton, Marvin Goodfriend, Charles Goodhart, Rafael Repullo and Ken Rogoff. We thank seminar participants at many universities as well as at the Bank of England, IMF, and the Richmond Fed.

# 1. Introduction

Our premise in this paper is that creditors face a coordination problem when facing a borrower in distress, and that this will be reflected in the price of debt. The problem faced by creditors is akin to that faced by depositors of a bank which is vulnerable to a run. Even if the project is viable, so that the value at maturity is enough to pay all the creditors in full, a creditor may be tempted to foreclose on the loan or seize any assets it can, fearing similar actions by other creditors. Such fears would be self-fulfilling, since the disorderly liquidation of assets and the consequent disruption to the project is more likely to lead to failure of the project.

It is hard to overstate the importance of coordination failures. The recognition of this problem - known as the “common pool problem” among lawyers - lies at the heart of corporate bankruptcy provisions across the world, taking on its most elaborate form in the chapter 11 provisions of the U.S. bankruptcy code (Baird and Jackson (1990), Jackson (1986)). Also, coordination failure among creditors has been fingered by many commentators as the main culprit in the recent series of international financial crises. Both Fischer (1999) and Radelet and Sachs (1998) - whatever their other differences - attribute the Asian financial crisis of 1997 to coordination failure among creditors and other market participants.

Given the importance of this problem, it is incongruous that it has received such scant attention from the literature on asset pricing. The main difficulty in incorporating coordination failure in a pricing theory for debt is that coordination problems lead to multiple equilibria, in the manner of Diamond and Dybvig (1983). Without quantifiable information on the incidence of coordination failure, it is impossible to incorporate this into the *ex ante* price of the debt. In this respect, our aim in this paper can be achieved only if we can provide a theory which explains the incidence of cases where a solvent borrower is forced into failure. In the terminology of banking theory, we must first have a theory of *solvent* but *illiquid* borrowers. Since this task is perhaps the most pressing issue in the theory of banking, we cannot underestimate the size of the task. However, we give a solution to this problem here. In doing so, we are able to present a framework which extracts a unique outcome as a function of underlying fundamentals.

The financial market turbulence of 1998 has focused renewed attention on the problem of pricing debt, and have served as a reminder of some of the shortcomings in our current understanding of the topic. The late summer and autumn of 1998 were exceptionally turbulent times, and none more so than in the bond market. For corporate bonds, the yield spread over government bonds widened sharply, as did the spread between high and low grade corporate bonds<sup>1</sup>. Such spreads were

---

<sup>1</sup>For U.S. corporate bonds below investment grade, the yield spreads over U.S. Treasury

almost without precedent - at least outside recessionary periods. This widening of spreads was also accompanied by the virtual drying up of new issues of corporate bonds, especially those below investment grade.

Not all of these increases in spreads could be attributed to deteriorating credit quality, since the spreads between different classes of treasury securities also widened. However, there is mounting evidence that many borrowers underwent a sharp deterioration in credit quality, especially banks and other financial institutions (see Nickell, Perraudin and Varotto (1999)).

But there is a puzzle here. Even during the most turbulent periods in 1998, access to bank credit by firms was much less affected than their access to the bond market<sup>2</sup>. If there was such a sharp deterioration of credit quality for borrowers in the bond market, how did bank lending escape largely unscathed? The answer, we suggest, lies in the fact that bank lending to firms suffered less from the coordination failure among lenders. If a bank is the sole creditor to a firm, there is no other creditor to worry about. However, for projects which draw on many disparate lenders, such coordination problems will figure prominently in the thinking of all the lenders. Certainly, the small number of empirical studies of financial restructuring of firms under distress suggest that instances of disorderly liquidation and deviations from priority of debtors may play a significant role (see, for instance, Franks and Torous (1994)).

Even before the events of 1997/8, it would be fair to say that the theories underlying the pricing of defaultable debt had not enjoyed the same broad consensus of support nor the empirical success of other applications of asset pricing theory. A classic reference in the theory of the valuation of debt is Merton (1974), which models company asset value as a geometric Brownian motion, and assumes that bankruptcy occurs when asset value reaches some given fixed level relative to liabilities. Then, the price of debt can be obtained from option pricing techniques. More refined treatments of this approach include Leland (1994) - recognizing debt level as a decision by the firm - and Longstaff and Schwartz (1995) - which allows interest rate risk.

However, the empirical success of this approach has been mixed. One early study is Jones, Mason and Rosenfeld (1984), which uses data from 1975 to 1981 and finds that the actual observed prices of corporate bonds are below those predicted by the theory, and that the prediction error is larger for lower rated

---

bonds of comparable maturity widened from about 3.75% in early August to around 6% by mid-October - the highest since the collapse of the U.S. junk bond market in the early 1990s. For highest rated (Aaa) investment grade bonds, the spreads widened from around 0.9% in early August to around 1.5% in mid-October. For bonds rated Baa, the spreads rose from around 1.5% to 2.3%. See IMF (1998a)

<sup>2</sup>See, for instance, the series of articles in the *Financial Times* and the *Wall Street Journal* in the week of October 16th 1998.

bonds. For investment grade bonds, the error is around 0.5%, while for non-investment grade bonds, the error is much larger, at around 10%. Subsequent work has suggested that over-pricing is resilient to various refinements of the theory, and alternatives have been proposed<sup>3</sup>.

This has posed a dilemma for practitioners whose aim is to measure default risk using the Merton model. The choice is either to abandon the approach altogether, or introduce *ad hoc* features which violate the internal consistency of the model. The best known implementation of the Merton model (made popular by the consulting firm, KMV Corporation) is a procedure known as “benchmarking” (see Nickell, Perraudin and Varotto (1999)). In this procedure, the asset volatility of the borrower is estimated from stock price data on the assumption that default takes place according to the Merton model. However, the theory is then discarded when calculating the probability of default. In its place, the default levels for asset values are inferred from historical data on apparently similar firms. The resulting default trigger values are, in general, different from the theoretical default triggers given by the Merton model, and hence generates an internal inconsistency in the procedure.

One of the contributions of our paper is to explain how the default trigger levels for asset values actually *shift* as the underlying asset changes in value. It thus provides a coherent theoretical framework for addressing the probability of default. By explaining the incidence of coordination failure as a function of the underlying fundamentals and other relevant parameters, it is possible to supply the missing link between the asset value of the borrower and the value of the asset which is just low enough to trigger default. Once the incidence of coordination failure can be calculated, it is then a matter of evaluating the ex ante value of coordination failure and incorporating this risk into the price of debt. This framework allows us to address two issues of current debate - the proper use of value at risk (VaR) analysis, and the role of greater ‘transparency’ in preventing market turbulence.

Value at risk analysis attempts to quantify the potential impact on the value of a portfolio of shifts in the underlying state. However, the current state of the art does not make any explicit provision for coordination failure. By quantifying the impact of coordination failure, it is possible to formulate a framework for credit risk analysis which addresses the effects of coordination failure. To anticipate our key finding, we show that when the fundamental viability of a loan deteriorates, the coordination problem becomes more acute, so that the ex ante asset value of

---

<sup>3</sup>See Anderson and Sundaresan (1996), who suggest that shifts in bargaining power between the creditors and managers may explain the price anomaly. An alternative approach is to assume that default is an exogenous event which follows some hazard rate process. Then, the default risk is reflected in a higher discount rate. Duffie and Singleton (1999) develop this approach.

the loan falls more than proportionately to the deterioration of the fundamentals. We dub this additional effect the ‘coordination effect’. It reinforces the conventional effect in which a shift in the payoff distribution increases the weight of the left tail of the distribution.

“Transparency” has become a touchstone of the policy response following the recent financial crises. The notion of transparency is multi-faceted and touches on a wide range of issues such as accountability, legitimacy, and the efficacy of the legal infrastructure in enforcing contracts. However, there is one narrow interpretation of transparency which focuses on the provision of more accurate and timely information to market participants. The unstated premise is that the improved provision of information will mitigate coordination failure. One of the possibilities opened up by our framework is that we can subject this premise to more rigorous scrutiny. To anticipate our main conclusion, we find little to suggest that the provision of more accurate information, by itself, is sufficient to prevent crises. The effect of improved information on the efficiency of the outcome is ambiguous at best. This raises some important issues in the policy debate. When calling for improved transparency, it is important to be clear as to *how* the improved information will improve the outcome. The mere provision of information is unlikely to mitigate coordination failure. Rather, the institutional backdrop will be important in the way that transparency affects the market outcome.

## 2. The Model

A group of creditors are financing a project. Each creditor is small in that an individual creditor’s stake is negligible as a proportion of the whole. We index the set of creditors by the unit interval  $[0, 1]$ . At the end of its term, the project yields a liquidation value  $v$ , which is uncertain at the time of investment. The financing is undertaken via a standard debt contract. The face value of the repayment is  $L$ , and each creditor receives this full amount if the realized value of  $v$  is large enough to cover repayment of debt.

At an interim stage, before the final realization of  $v$ , the creditors have an opportunity to review their investment. The loan is secured on collateral, whose liquidation value is  $K^* < L$  if it is liquidated at the interim stage, but has the lower value  $K_*$  if it is liquidated following the project’s failure. Thus,

$$K_* < K^* < L$$

At the interim stage, each creditor has a choice of either rolling over the loan until the project’s maturity, or seizing the collateral and selling it for  $K^*$ . The value of the project at maturity depends on two factors - the underlying state  $\theta$ , and the degree of disruption caused to the project by the early liquidation by creditors.

Denoting by  $\ell$  the proportion of creditors who foreclose on the loan at the interim stage, the realized value of the project is given by

$$v(\theta, \ell) = \begin{cases} V & \text{if } z\ell \leq \theta \\ K_* & \text{if } z\ell > \theta \end{cases} \quad (2.1)$$

where  $V$  is a constant greater than  $L$ , and  $z > 0$  is a parameter which measures the severity of disruption caused by early liquidation.

By normalizing the payoffs so that  $L = 1$  and  $K_* = 0$ , the payoffs to a creditor are given by the following matrix, where  $\lambda \equiv (K^* - K_*) / (L - K_*)$ .

	Project succeeds	Project fails
Roll over loan	1	0
Foreclose on loan	$\lambda$	$\lambda$

The bold line in figure 1 depicts the payoff to a creditor arising from the loan when proportion  $\ell$  foreclose on the loan.

[Figure 1 here]

To avoid notational clutter, we assume that if rolling over the loan yields the same expected payoff as foreclosing on the loan, then a creditor prefers to foreclose. This assumption plays no substantial role.

If the creditors know the value of  $\theta$  perfectly before deciding on whether to roll over the loan, their optimal strategy can be analysed thus. If  $\theta > z$ , then it is optimal to continue with the project, irrespective of the actions of the other creditors. This is so, since even if every other creditor recalls the loan, the project yields enough to pay back the full face value of the loan (equal to 1). This is more than  $\lambda$ . Conversely, if  $\theta < 0$ , then it is optimal to foreclose on the loan irrespective of the actions of the others. Even if all other creditors roll over their loans, the project yields zero, which is less than  $\lambda$ .

When  $\theta$  lies in the interval  $(0, z)$ , there is a coordination problem among the creditors. We may think of this interval as the set of states in which a “credit event” has occurred, and in which the creditors are in a position to seize assets if they so chose. If all other creditors roll over their loan, then the payoff to rolling over the loan is 1, so that rolling over the loan to maturity yields more than the premature liquidation value  $\lambda$ . However, if everyone else recalls the loan, the payoff is  $0 < \lambda$ , so that early liquidation is optimal. This type of coordination problem among creditors is analogous to the bank run problem (Diamond and Dybvig (1983)), and leads to multiple equilibria in the simple perfect information



game in which creditors choose their actions when  $\theta$  is common knowledge<sup>4</sup>.

Based on the structure outlined above, we proceed to develop a model of credit under imperfect information. When the creditors make their initial investment, they know that  $\theta$  is normally distributed with mean  $y$ , and precision  $\alpha$  (that is, with variance  $1/\alpha$ ). At the interim stage, when each creditor decides on whether to roll over the loan, each creditor receives information concerning  $\theta$ , but this information is imperfect. Creditor  $i$  observes the realization of the noisy signal

$$x_i = \theta + \varepsilon_i \quad (2.2)$$

where  $\varepsilon_i$  is normally distributed with mean 0 and precision  $\beta$ . For  $i \neq j$ ,  $\varepsilon_i$  and  $\varepsilon_j$  are independent.

A *strategy* for creditor  $i$  is a decision rule which maps each realization of  $x_i$  to an action (i.e. to roll over the loan, or to foreclose on the loan prematurely). An *equilibrium* is a profile of strategies - one for each creditor - such that a creditor's strategy maximizes his expected payment conditional on the information available, when all other creditors are following the strategies in the profile.

We have noted that when  $\theta$  is observed perfectly (so that  $x_i = \theta$ ), there is more than one equilibrium. Indeed, there is an (uncountable) infinity of equilibria in this case. When  $\theta$  is observed imperfectly, there is a unique equilibrium provided that the noise  $\varepsilon_i$  is sufficiently small, as we now demonstrate.

### 3. Unique Equilibrium

When creditor  $i$  observes the realization of the signal  $x_i$ , his posterior distribution of  $\theta$  is normal with mean

$$\xi_i \equiv \frac{\alpha y + \beta x_i}{\alpha + \beta} \quad (3.1)$$

and precision  $\alpha + \beta$ . We can then state the following result.

**Theorem 1.** If  $\alpha/\sqrt{\beta} \leq \sqrt{2\pi}/z$ , there is a unique equilibrium. Conversely, if  $\alpha/\sqrt{\beta} > \sqrt{2\pi}/z$ , there is a value of  $\lambda$  such that there is more than one equilibrium.

In interpreting this result, it is useful to consider the ex ante distribution of  $\theta$  as public information concerning  $\theta$ , distinguishing it from the private signal  $x_i$ . When the precision of the private signal (given by  $\beta$ ) becomes large while fixing

---

<sup>4</sup>We do not have much to add to the debate on whether a secondary market will mitigate inefficiencies, except to note that any attempt to internalize the externalities are confronted by coordination/free-rider problems at a higher level. See Gertner and Scharfstein (1991).

the precision of the public signal, we are guaranteed a unique equilibrium. Conversely, when the private signal is *not* sufficiently informative, then multiplicity of equilibrium re-emerges for some parameter values of the problem. The critical level of the informativeness of the private signal can be given a characterization in terms of whether  $\alpha/\sqrt{\beta}$  is smaller or larger than  $\sqrt{2\pi}/z$ .

### 3.1. Preliminaries

Let us begin by considering the following hypothetical situation. Suppose there is some given level  $\xi$  of the posterior belief of the state  $\theta$  such that every creditor rolls over the loan if and only if his posterior belief is higher than  $\xi$ . Then consider the expected payoff of rolling over the loan when one's posterior belief is exactly equal to  $\xi$ . In other words, consider the expected payoff from rolling over the loan at the switching point. Denote this payoff by  $U(\xi)$ . Our result on uniqueness is a consequence of the following pair of results.

**Lemma 1.** If  $\xi$  solves  $U(\xi) = \lambda$ , then there is an equilibrium in which everyone employs the switching strategy around  $\xi$ . If there is a unique  $\xi$  which solves  $U(\xi) = \lambda$ , then there is no other equilibrium.

**Lemma 2.**  $U'(\xi) \geq 0$  for all  $\xi$  if and only if  $\alpha/\sqrt{\beta} \leq \sqrt{2\pi}/z$ .

The first lemma draws on recent analysis of games without common knowledge of payoffs<sup>5</sup>. In fact, we shall prove a much stronger result - namely, that if there is a unique solution to  $U(\xi) = \lambda$ , then the switching strategy around  $\xi$  is the only strategy which survives the iterated deletion of dominated strategies. The general structure of our model conforms to the class of supermodular games examined by Milgrom and Roberts (1990), and it is illuminating to see the uniqueness result in this light. The details are presented in the appendix.

As for the second lemma, we give the proof here. If  $\ell$  is determined by everyone following the switching strategy around  $\xi$ , what is the critical value of  $\theta$  for which the project succeeds? In other words, we want the  $\theta$  which solves

$$\theta = z\ell \tag{3.2}$$

From (3.1), the switching strategy around  $\xi$  entails rolling over the loan if and only if the private signal  $x$  is greater than

$$x(\xi, y) \equiv \frac{\alpha + \beta}{\beta} \xi - \frac{\alpha}{\beta} y. \tag{3.3}$$

---

<sup>5</sup>The work of Rubinstein (1989) and Monderer and Samet (1989) was followed by Carlsson and Van Damme (1993a, b), who introduced the notion of “global games”. Morris, Rob and Shin (1995), and Kajii and Morris (1997) develop these results. Morris and Shin (1997) is a survey of some of the early results. Morris and Shin (1998a, b) apply these results in the analysis of currency attacks.

Conditional on state  $\theta$ , the distribution of  $x$  is normal with mean  $\theta$  and precision  $\beta$ . Hence, the proportion of creditors who have a signal lower than (3.3) is given by the area under this density up to  $x$ . Hence,

$$\ell = \Phi \left( \sqrt{\beta} (x(\xi, y) - \theta) \right) \quad (3.4)$$

where  $\Phi(\cdot)$  is the cumulative distribution function for the standard normal. Substituting into (3.2), we have an expression for the critical value of  $\theta$  at which the project succeeds. That is,

$$\theta = z\Phi \left( \sqrt{\beta} (x(\xi, y) - \theta) \right) \quad (3.5)$$

This value of  $\theta$  is unique<sup>6</sup>, and is a function of  $\xi$  and  $y$ , and so we write

$$\psi(\xi, y) \quad (3.6)$$

as the unique value of  $\theta$  which solves (3.5).  $\psi(\xi, y)$  satisfies

$$\psi(\xi, y) = z\Phi \left( \sqrt{\beta} (x(\xi, y) - \psi(\xi, y)) \right) \quad (3.7)$$

Now, the payoff  $U(\xi)$  is given by

$$\begin{aligned} U(\xi) &= \int_{\psi}^{\infty} \phi \left( \sqrt{\alpha + \beta} (\theta - \xi) \right) d\theta \\ &= 1 - \Phi \left( \sqrt{\alpha + \beta} (\psi - \xi) \right) \end{aligned} \quad (3.8)$$

so that

$$U'(\xi) = -\sqrt{\alpha + \beta} \cdot \phi \left( \sqrt{\alpha + \beta} (\psi - \xi) \right) \cdot \left( \frac{\partial \psi}{\partial \xi} - 1 \right) \quad (3.9)$$

where  $\phi(\cdot)$  is the density of the standard normal. Hence  $U'(\xi) \geq 0$  if and only if  $\frac{\partial \psi}{\partial \xi} \leq 1$ . By implicit differentiation of (3.7) with respect to  $\xi$ ,

$$\frac{\partial \psi}{\partial \xi} = z\sqrt{\beta} \left( \frac{\alpha + \beta}{\beta} - \frac{\partial \psi}{\partial \xi} \right) \phi \left( \sqrt{\beta} (x - \psi) \right)$$

Solving for  $\frac{\partial \psi}{\partial \xi}$ , we have

$$\frac{\partial \psi}{\partial \xi} = \frac{\phi\sqrt{\beta}}{\frac{1}{z} + \phi\sqrt{\beta}} \cdot \frac{\alpha + \beta}{\beta}$$

---

<sup>6</sup>The uniqueness follows from the fact that the right hand side is decreasing in  $\theta$ , continuous, and takes all values in the open interval  $(0, 1)$ . Hence there is a unique point at which it cuts the 45° line.

Thus,  $\frac{\partial \psi}{\partial \xi} \leq 1$  if and only if

$$\frac{\phi \sqrt{\beta}}{\frac{1}{z} + \phi \sqrt{\beta}} \leq \frac{\beta}{\alpha + \beta} \quad (3.10)$$

Since the left hand side is maximized at  $\phi = 1/\sqrt{2\pi}$ , a sufficient condition for  $\frac{\partial \psi}{\partial \xi} \leq 1$  is  $\sqrt{\beta}/\left(\frac{\sqrt{2\pi}}{z} + \sqrt{\beta}\right) \leq \beta/(\alpha + \beta)$  which boils down to  $\alpha/\sqrt{\beta} \leq \sqrt{2\pi}/z$ . Conversely, suppose  $\alpha/\sqrt{\beta} > \sqrt{2\pi}/z$ . Then, from (3.10),  $\frac{\partial \psi}{\partial \xi} > 1$  when  $x = \psi$ . This proves the lemma.

## 4. Default

Although theorem 1 tells us when equilibrium is unique, we would like more. To answer questions concerning the comparative statics of the equilibrium, we need to solve explicitly for the equilibrium. Solving for the unique equilibrium entails solving, first, for the switching point  $\xi$  (which lies in the space of posterior beliefs) and for the failure point  $\psi$ , which lies in the space of fundamentals. Clearly, one depends on the other. The critical state at which the project fails depends on the strategies used by the creditors, while a strategy used by a creditor will take into account where the project fails. Thus, we must solve for the failure point  $\psi$  and switching point  $\xi$  simultaneously. We do this by extracting two equations involving  $\psi$  and  $\xi$ .

The critical state at which the project fails is the state  $\theta$  for which  $\theta = z\ell$ , where  $\ell$  is generated by the equilibrium switching strategy. From (3.7), the critical state  $\psi$  satisfies the equation:

$$\begin{aligned} \psi &= z\Phi\left(\sqrt{\beta}(x - \psi)\right) \\ &= z\Phi\left(\sqrt{\beta}\left(\frac{\alpha + \beta}{\beta}\xi - \frac{\alpha}{\beta}y - \psi\right)\right) \\ &= z\Phi\left(\frac{\alpha}{\sqrt{\beta}}(\xi - y) + \sqrt{\beta}(\xi - \psi)\right) \end{aligned} \quad (4.1)$$

This gives us our first equation in terms of  $\xi$  and  $\psi$ .

For our second equation, we appeal to the fact that the switching point  $\xi$  is the unique solution to  $U(\xi) = \lambda$ . In other words,

$$1 - \Phi\left(\sqrt{\alpha + \beta}(\psi - \xi)\right) = \lambda \quad (4.2)$$

which implies

$$\psi - \xi = \frac{\Phi^{-1}(1 - \lambda)}{\sqrt{\alpha + \beta}}. \quad (4.3)$$

This gives us our second equation. From this pair of equations, we can solve for our two unknowns,  $\psi$  and  $\xi$ . Solving for  $\psi$ , we have

$$\psi = z\Phi\left(\frac{\alpha}{\sqrt{\beta}}\left(\psi - y + \Phi^{-1}(\lambda)\frac{\sqrt{\alpha+\beta}}{\alpha}\right)\right). \quad (4.4)$$

The failure point  $\psi$  is obtained as the intersection between the 45° line and a scaled-up cumulative normal distribution whose mean is  $y - \Phi^{-1}(\lambda)\frac{\sqrt{\alpha+\beta}}{\alpha}$ , and whose precision is  $\alpha^2/\beta$ . From theorem 1, we know that there is precisely one point of intersection, since equilibrium is unique<sup>7</sup>.

[Figure 2 here]

When  $z$  is large, the destruction of value can be very substantial. The interval  $[0, \psi]$  represents the size of the inefficiency. These are the states at which liquidation is inefficient, but liquidation is forced on the borrower. To use the terminology of banking theory, the size of the interval  $[0, \psi]$  measures the incidence of cases where the borrower is solvent, but illiquid.

The failure point  $\psi$  depends on the parameters of the problem. We note that

- $\psi$  is increasing in  $\lambda$
- $\psi$  is increasing in  $z$
- $\psi$  is decreasing in  $y$ , the ex ante mean of  $\theta$ .

Thus, failure occurs at higher values of the fundamentals if the collateral has high liquidation value, when the success of the project is more fragile to the early liquidation of creditors, and when the debt is of low quality. The fact that failure is more likely when  $y$  is low is an important result, and we turn to this now.

#### 4.1. Value at Risk

Let us begin by verifying that the failure point  $\psi$  does indeed move in the opposite direction to  $y$ . Differentiate (4.4) to obtain

$$\frac{\partial\psi}{\partial y} = z\frac{\alpha}{\sqrt{\beta}}\left(\frac{\partial\psi}{\partial y} - 1\right)\phi$$

so that

$$\frac{\partial\psi}{\partial y} = -\frac{\frac{\alpha}{\sqrt{\beta}}z\phi}{1 - \frac{\alpha}{\sqrt{\beta}}z\phi} \quad (4.5)$$

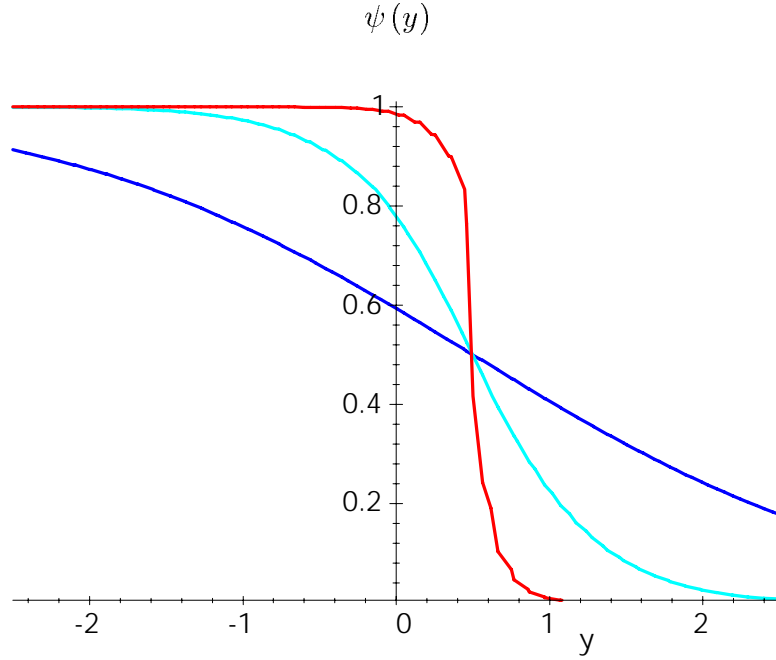
---

<sup>7</sup>This is reflected in (4.4) by the fact that the slope of the expression on the right is less than one when  $\alpha/\sqrt{\beta} \leq \sqrt{2\pi}/z$ .

But the denominator is positive, since uniqueness implies (by Theorem 1)  $\alpha/\sqrt{\beta} \leq \sqrt{2\pi}/z$ , and  $\phi$  is bounded above by  $1/\sqrt{2\pi}$ . Hence,

$$\frac{\partial \psi}{\partial y} < 0.$$

Consider a numerical example with  $\lambda = 1/2$  and  $z = 1$  where we plot the failure point  $\psi$  as a function of  $y$ . The largest value of  $\alpha/\sqrt{\beta}$  which ensures uniqueness is  $\sqrt{2\pi} \approx 2.5$ . The function  $\psi(y)$  for this value is given by the steepest line in figure 4. The figure also plots  $\psi$  for  $\alpha/\sqrt{\beta}$  taking the values 1 and 0.4. As  $\alpha/\sqrt{\beta} \rightarrow 0$ ,  $\psi(y)$  tends to the constant function passing through  $1/2$ .



Plots of  $\psi(y)$  when  $\alpha/\sqrt{\beta}$  takes values 2.5, 1 and 0.4

[Figure 3]

As a comparison, it should be borne in mind that the benchmark model which ignores coordination risk is equivalent to assuming that the failure point  $\psi$  is the constant function through zero.

The fact that the failure point moves up as the fundamentals deteriorate has far-reaching consequences for risk management. For example, consider the value of an *unsecured* loan to the project with face value 1. The owner of such an asset

only receives a positive payoff when the true state is higher than  $\psi$ . The ex ante price of such a loan given ex ante mean  $y$  is

$$W(y) = \int_{\psi}^{\infty} \phi(\sqrt{\alpha}(\theta - y)) d\theta = 1 - \Phi(\sqrt{\alpha}(\psi - y)) \quad (4.6)$$

For the owner of this asset who wishes to calculate the possible changes in price, one important consideration is how  $\psi$  changes with shifts in the ex ante mean  $y$  of the project. Value at risk analysis can be seen as an attempt to quantify the possible changes in price as  $y$  changes. The change in the asset value of the loan to a shift in the mean of the distribution is

$$\frac{\partial W}{\partial y} = \sqrt{\alpha} \cdot \phi - \frac{\partial \psi}{\partial y} \sqrt{\alpha} \cdot \phi \quad (4.7)$$

The first term could be dubbed the *conventional effect* in that it reflects the change in the weight of the left tail of the distribution due to a shift in the centre of the distribution. The second term is the novel feature. It arises from the fact that the threshold for the tail also shifts. We could call this the *coordination effect*. Since  $\partial \psi / \partial y < 0$ , the coordination effect reinforces the conventional effect. Figure 3 illustrates the two effects.

[Figure 4 here]

As a function of  $y$ , the critical value  $\psi$  is decreasing in  $y$ . As the fundamentals deteriorate from  $y$  to  $\hat{y}$ , the critical value of the state shifts to the *right* from  $\psi$  to  $\hat{\psi}$ . Thus, the asset value of the loan falls for two reasons. For a fixed threshold, the distribution puts more weight on the lower tail. This is the conventional effect, and is indicated as the area A. The second effect arises from the fact that the critical threshold moves up, also. This is the coordination effect, and is indicated by the area B.

For the creditor, a deterioration in the fundamentals in terms of a fall in  $y$  implies that the asset value of the loan is falling at a rate *more than proportional* to the fall in  $y$ . Thus, it is precisely when risk management is most important - when  $y$  is falling - that it is important not to neglect the coordination effect. By neglecting the coordination effect, the creditor is underestimating the true value at risk.

Note also that such an effect is exactly what is needed to rescue the Merton model for defaultable debt. The greater incidence of coordination failure for lower quality debt implies a higher default trigger. We can illustrate this by means of the following numerical example.

## 4.2. Numerical Example

Defining the yield on the unsecured loan as

$$\text{Yield} = \frac{\text{Par} - \text{Price}}{\text{Price}},$$

we can compare the yields generated by the true model (with failure occurring at  $\psi$ ) with the yields given by the naive model which assumes away coordination risk. The following table is generated from the case where

$$\alpha = 1, \quad \beta = 5, \quad z = 1, \quad \lambda = 0.5.$$

The first column gives the ex ante mean of the payoff distribution and the second column gives the yield on the loan for the naive model (no coordination risk). The third column gives the yield arising from the true model, and the value of the break point  $\psi(y)$  appears in the last column. Since  $\alpha = 1$ , the values of  $y$  are in units of standard deviations. So, the first row of the table pertains to the case where the ex ante mean  $y$  is three standard deviations from zero. The last element of this row tells us that the true failure point is  $\psi(3) = 0.097$ , and the true yield is 0.19%, rather than the yield given by the naive model of 0.14%. This difference in yield is not large, since the loan is a very safe one - the mean being three standard deviations away from zero.

Ex ante mean $y$	Yield from naive model	Yield from true model	Failure point $\psi$
$y = 3$	0.0014	0.0019	0.097
$y = 2$	0.0233	0.0383	0.212
$y = 1.5$	0.0716	0.1288	0.295
$y = 1.25$	0.1181	0.2226	0.342
$y = 1$	0.1886	0.3735	0.393
$y = 0.75$	0.293	0.6143	0.446
$y = 0.5$	0.4462	1.0	0.5
$y = 0.25$	0.6703	1.6279	0.554
$y = 0$	1.0	2.6774	0.607

However, as  $y$  falls, we can see that the yield difference becomes large. At one standard deviation away from zero (i.e. for  $y = 1$ ), the naive model predicts a yield of 19%, but the true yield is actually almost double that number, at 37%.



This corresponds to the break point of  $\psi = 0.393$ . Thus, the interval  $[0, 0.393]$  represents the size of inefficient liquidation. For even lower values of  $y$ , the yield difference is even higher. When  $y = 0$ , the naive model predicts a yield of 100%, but the true yield is 268%.

Such a pattern of discrepancies between the benchmark model and the true model is quite suggestive. The overpricing of defaultable bonds relative to market prices (and the underpricing of its yield), as well as the fact that such discrepancies are larger for lower quality bonds, has been one of the persistent problems with empirical implementations of the Merton model. Our theory predicts that the default point will actually be a function of the quality of the bond, and that the default point will be higher for lower quality bonds. When this additional effect is taken into account, the apparent anomalies can be accommodated. It would appear that this feature provides an appropriate remedy for the apparent failure of the Merton model.

Furthermore, although the parameters  $\alpha$  and  $\beta$  are rather abstract quantities pertaining to information, they have an indirect empirical counterpart in terms of the relationship between the fundamental value  $y$  and the failure point  $\psi$ . Thus, in principle, it would be possible to extract some information on  $\alpha$  and  $\beta$  if we had sufficiently detailed data on the relationship between the distance to default and the default probability.

## 5. Transparency

The detrimental effect of imperfect information on the asset value of the loan can be sizeable. It is therefore pertinent to ask whether, and by how much, the damage can be limited by improvements in the information of the market participants. The term “transparency” has taken on great significance in the policy debate after the market turmoil of 1997/8, and has figured prominently in numerous official publications (such as IMF (1998b), BIS (1999)). The notion of transparency has many subtleties, and it would be wrong to give too simplistic an interpretation of it. However, it is instructive to examine one particular interpretation of this notion purely in terms of the provision of more accurate and timely information to market participants. The unstated premise in the call for more transparency is that the improved provision of information will enable market participants to act in such a way that the destruction of value through imperfect coordination can be minimized.

Having developed the framework so far, we are now in a position to subject this premise to more rigorous scrutiny. To anticipate our main conclusion, we find little to suggest that the provision of more accurate information (by itself) is sufficient to improve matters. The effect of improved information on the efficiency

of the outcome is ambiguous at best.

We consider two ways in which information improves. In the first, we envisage the creditors' private information as being very accurate relative to the underlying uncertainty on the state  $\theta$ . We can formalize this in terms of the precision  $\beta$  of the private signals become infinite while  $\alpha$  remains fixed.

In the second, we examine a rather different formalization of the improvement in information. Here, we envisage the quality of the *public information* improving without bound (i.e.) of the precision  $\alpha$  becoming large. However, we take into consideration the fact that improved public information will provide the necessary platform for even more precise private information. For instance, if the government or the central bank were to provide more information to the market, this is more grist to the mill for the research departments of the numerous banks and other financial institutions who will generate yet more private information based on such disclosures.

Specifically, we consider the case where both  $\alpha$  and  $\beta$  become large, but where  $\beta$  increases at the rate of  $\alpha^2$ . Formally, this is also necessary in order for us to keep the uniqueness of equilibrium at every point in the sequence.

### 5.1. Case of Precise Private Information

When the noise in the creditors' signals become small, each creditor has good information about the underlying state  $\theta$ . What happens in the limit when the noise becomes negligible? This corresponds to the case where the precision  $\beta$  becomes large relative to  $\alpha$ . From (4.1) and (4.2), the limit of  $\xi$  when  $\beta \rightarrow \infty$  is:

$$\xi = z\Phi(\Phi^{-1}(\lambda)) = z\lambda \quad (5.1)$$

Since  $\psi = \xi$  in the limit, the critical state  $\psi$  is also given by  $z\lambda$ . For large  $z$ , the efficiency loss is sizeable. Nor is there any reason to suppose that this efficiency loss is smaller than in typical cases with positive noise. In fact, figure 3 suggests that the effect can be perverse, depending on the parameter values. Indeed, for the more plausible case where  $y$  lies to the right of  $\lambda z$ , an increase in  $\beta$  "flattens" the failure schedule, pushing up the failure point. Thus, more information leads to a greater incidence of coordination failure.

However, one consequence of an infinite  $\beta$  is the fact that the critical state  $\psi$  no longer depends on the prior mean  $y$ , so that the "coordination effect" of value at risk disappears. At face value, this is quite natural, since as  $\beta$  becomes large, the information content of the private signal swamps the information contained in the prior distribution. We say "at face value", as such reasoning can be quite treacherous. Indeed, we will see one instance of this in our second formulation of transparency.

## 5.2. Case of Precise Private and Public Information

One element emphasized by those advocating greater transparency in financial markets is the timely provision of official statistics and other market related information in a public forum. Timely provision has to do with frequent and up to date data for public scrutiny. The goal of such dissemination would be to increase the precision  $\alpha$  of the ex ante distribution, so as to reduce the overall uncertainty facing the market. However, since market participants will have access to other information in addition to such public information, we must regard the precision  $\beta$  of the private information as increasing at a faster rate. Here, we let both  $\alpha$  and  $\beta$  tend to infinity, but keep the ratio  $\alpha/\sqrt{\beta}$  constant - where the constant is small enough to guarantee uniqueness of equilibrium. Thus, consider a sequence of pairs  $(\alpha, \beta)$  where, for constant  $c$ ,

$$\alpha \rightarrow \infty, \quad \beta \rightarrow \infty, \quad \text{but} \quad \frac{\alpha}{\sqrt{\beta}} = c \leq \frac{\sqrt{2\pi}}{z}$$

Then,  $\sqrt{\beta/(\alpha + \beta)} = 1/\sqrt{1 + c/\sqrt{\beta}} \rightarrow 1$ , so that the limit of  $\psi$  is

$$\psi = z\Phi\left(c\left(\psi - y + \frac{\Phi^{-1}(\lambda)}{c}\right)\right) \quad (5.2)$$

Notice the re-appearance of the ex ante mean  $y$  in this expression. Even though the private signal swamps the public information, the ex ante mean is still relevant in determining the critical state  $\psi$ . Thus, the “coordination effect” in value at risk returns with a vengeance. Greater transparency in terms of better public (and private) information also has little obvious effect in promoting overall efficiency (compare (5.2) with (4.4)). At first, this is somewhat puzzling, since the information contained in the public signal ought to be dominated by the more accurate private signal. However, this is to neglect the information contained in  $y$  concerning the beliefs of *other* creditors. The ex ante mean  $y$  is relevant not because of the information conveyed about the fundamentals, but rather because it conveys information about the distribution of other creditors’ beliefs, and it is this which is crucial in strategic situations such as ours.

This last observation holds some important lessons for the conduct of public policy in dissemination of information. When calling for improved transparency, it is important to be clear as to *how* the improved information will improve the outcome. The mere provision of information will not be enough. However, if the improved information is one element of better coordination of the disparate market participants, the information may have some beneficial effect. This suggests that concrete institutional changes must accompany the provision of information if the information is to be effective.

In spite of the acknowledged simplicity of the model, we may nevertheless draw some lessons for the current debate concerning the reform of the international financial system. With the benefit of theoretical hindsight, it is perhaps not surprising that the provision of more information to market participants does not mitigate the problem. After all, we should draw a distinction between a single-person decision problem and a strategic situation. In a single-person decision problem, more information is always more valuable. When I debate whether to carry an umbrella into work, an accurate weather forecast will minimize both the inconvenience of carrying a bulky umbrella on a sunny day, and also the opposite inconvenience of getting caught in a shower without shelter. In such instances, “transparency” works.

However, it is far from clear whether better information will mitigate a coordination problem. There is little guidance from economic theory that better information about payoffs to players of a coordination game leads to greater incidence of successful coordination. Indeed, the intuition conveyed by existing theory is of a much more prosaic sort - typified by the debate on the Coase Theorem - in which all the emphasis is placed on the impediments to efficient bargaining. When the interested parties are diffuse and face uncertainty both about the fundamentals and the information of others, it would be overly optimistic to expect *ex post* efficient bargains to be struck.

We have already noted how instances of successful coordination by creditors - such as the bailout of Long Term Capital Management in September 1998 - have had a forceful facilitator organizing the bailout. In the case of LTCM, this role was played by the New York Fed. The U. S. Treasury has also played a key role in a number of episodes in recent years (Brazil in 1999, Korea in 1997/8). Although governments and central banks are best placed to play such a role, there is no reason why a non-governmental party cannot play a similar role. The account of J. P. Morgan’s role in coordinating the 1907 bailout is an instructive example<sup>8</sup>. Proponents of more elaborate multilateral institutions would do well to pause for thought on how the new institution will fare in the role of facilitator.

## Appendix

In this appendix, we provide an argument for lemma 1. In fact, we can show a much stronger result - namely, that if there is a unique solution to  $U(\xi) = \lambda$ , then

---

<sup>8</sup>See, for instance, New Yorker magazine, Nov 23, 1998 (page 62). We are grateful for Arijit Mukherji for this reference.

the switching strategy around  $\xi$  is not only the unique *equilibrium* strategy, it is also the only strategy which survives the iterated deletion of dominated strategies.

Consider first the expected payoff to rolling over the loan conditional on  $\xi$  when all others are using the switching strategy around some point  $\hat{\xi}$ . Denote this payoff as

$$u(\xi, \hat{\xi}) \quad (5.3)$$

This payoff is given by  $1 - \Phi(\sqrt{\alpha + \beta}(\psi - \xi))$ , where  $\psi$  is the failure point defined as the unique solution to the equation  $\psi = z\Phi(\sqrt{\beta}(x(\hat{\xi}, y) - \psi))$ . The conditional payoff  $u(\xi, \hat{\xi})$  can be seen to satisfy the following three properties.

**Monotonicity.**  $u$  is strictly increasing in its first argument, and is strictly decreasing in its second argument.

**Continuity.**  $u$  is continuous.

**Full Range.** For any  $\hat{\xi} \in \mathbb{R} \cup \{-\infty, \infty\}$ ,  $u(\xi, \hat{\xi}) \rightarrow 0$  as  $\xi \rightarrow -\infty$ , and  $u(\xi, \hat{\xi}) \rightarrow 1$  as  $\xi \rightarrow \infty$ .

By appealing to these features, we can define two sequences of real numbers. First, define the sequence

$$\underline{\xi}^1, \underline{\xi}^2, \dots, \underline{\xi}^k, \dots \quad (5.4)$$

as the solutions to the equations:

$$\begin{aligned} u(\underline{\xi}^1, -\infty) &= \lambda \\ u(\underline{\xi}^2, \underline{\xi}^1) &= \lambda \\ &\vdots \\ u(\underline{\xi}^{k+1}, \underline{\xi}^k) &= \lambda \\ &\vdots \end{aligned}$$

In an analogous way, we define the sequence

$$\bar{\xi}^1, \bar{\xi}^2, \dots, \bar{\xi}^k, \dots \quad (5.5)$$

as the solutions to the equations:

$$\begin{aligned} u(\bar{\xi}^1, \infty) &= \lambda \\ u(\bar{\xi}^2, \bar{\xi}^1) &= \lambda \end{aligned}$$

$$\begin{array}{c}
\vdots \\
u(\bar{\xi}^{k+1}, \bar{\xi}^k) = \lambda \\
\vdots
\end{array}$$

We can then prove:

**Lemma A1.** Let  $\xi$  solve  $U(\xi) = \lambda$ . Then

$$\underline{\xi}^1 < \underline{\xi}^2 < \dots < \underline{\xi}^k < \dots < \xi \quad (5.6)$$

$$\bar{\xi}^1 > \bar{\xi}^2 > \dots > \bar{\xi}^k > \dots > \xi \quad (5.7)$$

Moreover, if  $\underline{\xi}$  and  $\bar{\xi}$  are, respectively, the smallest and largest solutions to  $U(\xi) = \lambda$ , then

$$\underline{\xi} = \lim_{k \rightarrow \infty} \underline{\xi}^k \quad \text{and} \quad \bar{\xi} = \lim_{k \rightarrow \infty} \bar{\xi}^k. \quad (5.8)$$

**Proof.** Since  $u(\xi^1, -\infty) = u(\xi^2, \xi^1) = \lambda$ , monotonicity implies  $\xi^1 < \xi^2$ . Thus, suppose  $\xi^{k-1} < \xi^k$ . Since  $u(\xi^k, \xi^{k-1}) = u(\xi^{k+1}, \xi^k) = \lambda$ , monotonicity implies  $\xi^k < \xi^{k+1}$ . Finally, since  $U(\xi) = u(\xi, \xi) = u(\xi^{k+1}, \xi^k)$ , and  $\xi^k < \xi^{k+1}$ , monotonicity implies that  $\xi^k < \xi$ . Thus,  $\xi^1 < \xi^2 < \dots < \xi^k < \dots < \xi$ . An exactly analogous argument shows that  $\bar{\xi}^1 > \bar{\xi}^2 > \dots > \bar{\xi}^k > \dots > \xi$ . Now, suppose  $\underline{\xi}$  is the smallest solution to  $u(\xi, \xi) = \lambda$ . By (5.6) and the monotonicity of  $u$ ,  $\underline{\xi}$  is the smallest upper bound for the sequence  $\{\xi^k\}$ . Since  $\{\xi^k\}$  is an increasing, bounded sequence, it converges to its smallest upper bound. Thus  $\underline{\xi} = \lim_{k \rightarrow \infty} \xi^k$ . Analogously, if  $\bar{\xi}$  is the largest solution to  $u(\xi, \xi) = \lambda$ , then (5.7) and monotonicity of  $u$  implies that  $\bar{\xi} = \lim_{k \rightarrow \infty} \bar{\xi}^k$ . This proves the lemma.

**Lemma A2.** If  $\sigma$  is a strategy which survives  $k$  rounds of iterated deletion of interim dominated strategies, then

$$\sigma(\xi) = \begin{cases} F & \text{if } \xi < \underline{\xi}^k \\ R & \text{if } \xi > \bar{\xi}^k \end{cases} \quad (5.9)$$

The argument is as follows. Let  $\sigma^{-i}$  be the strategy profile used by all players other than  $i$ , and denote by  $\tilde{u}^i(\xi, \sigma^{-i})$  the payoff to  $i$  of rolling over the loan conditional on  $\xi$  when the others' strategy profile is given by  $\sigma^{-i}$ . The incidence of failure is minimized when everyone is rolling over the loan irrespective of the

signal, and the the incidence of failure is *maximized* when everyone is foreclosing on the loan irrespective of the signal. Thus, for any  $\xi$  and any  $\sigma^{-i}$ ,

$$u(\xi, \infty) \leq \tilde{u}^i(\xi, \sigma^{-i}) \leq u(\xi, -\infty) \quad (5.10)$$

From the definition of  $\underline{\xi}^1$  and monotonicity,

$$\xi < \underline{\xi}^1 \implies \text{for any } \sigma^{-i}, \tilde{u}^i(\xi, \sigma^{-i}) \leq u(\xi, -\infty) < u(\underline{\xi}^1, -\infty) = \lambda. \quad (5.11)$$

In other words,  $\xi < \underline{\xi}^1$  implies that rolling over the loan is strictly dominated by foreclosing. Similarly, from the definition of  $\bar{\xi}^1$  and monotonicity,

$$\xi > \bar{\xi}^1 \implies \text{for any } \sigma^{-i}, \tilde{u}^i(\xi, \sigma^{-i}) \geq u(\xi, \infty) > u(\bar{\xi}^1, \infty) = \lambda. \quad (5.12)$$

In other words,  $\xi > \bar{\xi}^1$  implies that foreclosing on the loan is strictly dominated by rolling over. Thus, if strategy  $\sigma^i$  survives the initial round of deletion of dominated strategies,

$$\sigma^i(\xi) = \begin{cases} F & \text{if } \xi < \underline{\xi}^1 \\ R & \text{if } \xi > \bar{\xi}^1 \end{cases} \quad (5.13)$$

so that (5.9) holds for  $k = 1$ .

For the inductive step, suppose that (5.9) holds for  $k$ , and denote by  $U^k$  the set of strategies which satisfy (5.9) for  $k$ . We must now show that, if player  $i$  faces a strategy profile consisting of those drawn from  $U^k$ , then any strategy which is not in  $U^{k+1}$  is dominated. Thus, suppose that player  $i$  believes that he faces a strategy profile  $\sigma^{-i}$  consisting of strategies from  $U^k$ . Given this, the incidence of failure is minimized when  $\sigma^{-i}$  is the (constant) profile consisting of the  $\bar{\xi}^k$ -trigger strategy, and the the incidence of failure is *maximized* when  $\sigma^{-i}$  is the (constant) profile consisting of  $\underline{\xi}^k$ -trigger strategy. Thus, for any  $\xi$  and any strategy profile  $\sigma^{-i}$  consisting of those from  $U^k$ ,

$$u(\xi, \bar{\xi}^k) \leq \tilde{u}^i(\xi, \sigma^{-i}) \leq u(\xi, \underline{\xi}^k) \quad (5.14)$$

From the definition of  $\underline{\xi}^k$  and monotonicity, we have the following implication for any strategy profile  $\sigma^{-i}$  drawn from  $U^k$ .

$$\xi < \underline{\xi}^{k+1} \implies \tilde{u}^i(\xi, \sigma^{-i}) \leq u(\xi, \underline{\xi}^k) < u(\underline{\xi}^{k+1}, \underline{\xi}^k) = \lambda. \quad (5.15)$$

In other words, when  $\xi < \underline{\xi}^k$  and when all others are using strategies from  $U^k$ , rolling over the loan is strictly dominated by foreclosing. Similarly, from the definition of  $\bar{\xi}^k$  and monotonicity, we have the following implication for any strategy profile  $\sigma^{-i}$  consisting of those from  $U^k$ .

$$\xi > \bar{\xi}^{k+1} \implies \tilde{u}^i(\xi, \sigma^{-i}) \geq u(\xi, \bar{\xi}^k) > u(\bar{\xi}^{k+1}, \bar{\xi}^k) = \lambda. \quad (5.16)$$

In other words, when  $\xi > \bar{\xi}^{k+1}$  and all others are using strategies from  $U^k$ , foreclosing on the loan is strictly dominated by rolling over. Thus, if strategy  $\sigma^i$  survives  $k + 1$  rounds of iterated deletion of dominated strategies,

$$\sigma^i(\xi) = \begin{cases} F & \text{if } \xi < \bar{\xi}^{k+1} \\ R & \text{if } \xi > \bar{\xi}^{k+1} \end{cases} \quad (5.17)$$

This proves the lemma.

With these preliminary results, we can complete the proof of Lemma 1. First, let us show that if  $\xi$  solves  $U(\xi) = \lambda$ , then there is an equilibrium in trigger strategies around  $\xi$ . Since  $U(\xi) = u(\xi, \xi) = \lambda$ , if everyone else is using the  $\xi$ -trigger strategy, the payoff to rolling over conditional on  $\xi$  is the same as that for foreclosing. Since  $u$  is strictly increasing in its first argument,

$$\xi_* < \xi < \xi^* \iff u(\xi_*, \xi) < \lambda < u(\xi^*, \xi)$$

so that the  $\xi$ -trigger strategy is the strict best reply.

Finally, let us show that if  $\xi$  is the unique solution to  $U(\xi) = \lambda$ , then there is no other equilibrium. From Lemma A1, we know that

$$\xi = \lim_{k \rightarrow \infty} \underline{\xi}^k = \lim_{k \rightarrow \infty} \bar{\xi}^k \quad (5.18)$$

so that the only strategy which survives the iterated deletion of dominated strategies is the  $\xi$ -trigger strategy. Among other things, this implies that the equilibrium in  $\xi$ -trigger strategies is the unique equilibrium.

The basic properties of our model conform to the class of supermodular games examined by Milgrom and Roberts (1990), in that the payoffs exhibit strategic complementarities, and the strategy set can be seen as a lattice for the appropriate ordering of strategies. The following features of our model echo the general results obtained by Milgrom and Roberts.

- There is a “smallest” and “largest” equilibrium, corresponding to the smallest and largest solutions to the equation  $U(\xi) = \lambda$ .
- Any strategy other than those lying between the smallest and largest equilibrium strategies can be eliminated by iterated deletion of dominated strategies. Thus, if  $\underline{\xi}$  and  $\bar{\xi}$  are, respectively, the smallest and largest solutions to  $U(\xi) = \lambda$ , then rationalizability removes all indeterminacy in a player’s strategy except for the interval  $[\underline{\xi}, \bar{\xi}]$ .
- If there is a unique solution to  $U(\xi) = \lambda$ , then there is a unique equilibrium, and this is obtained as the uniquely rationalizable strategy.

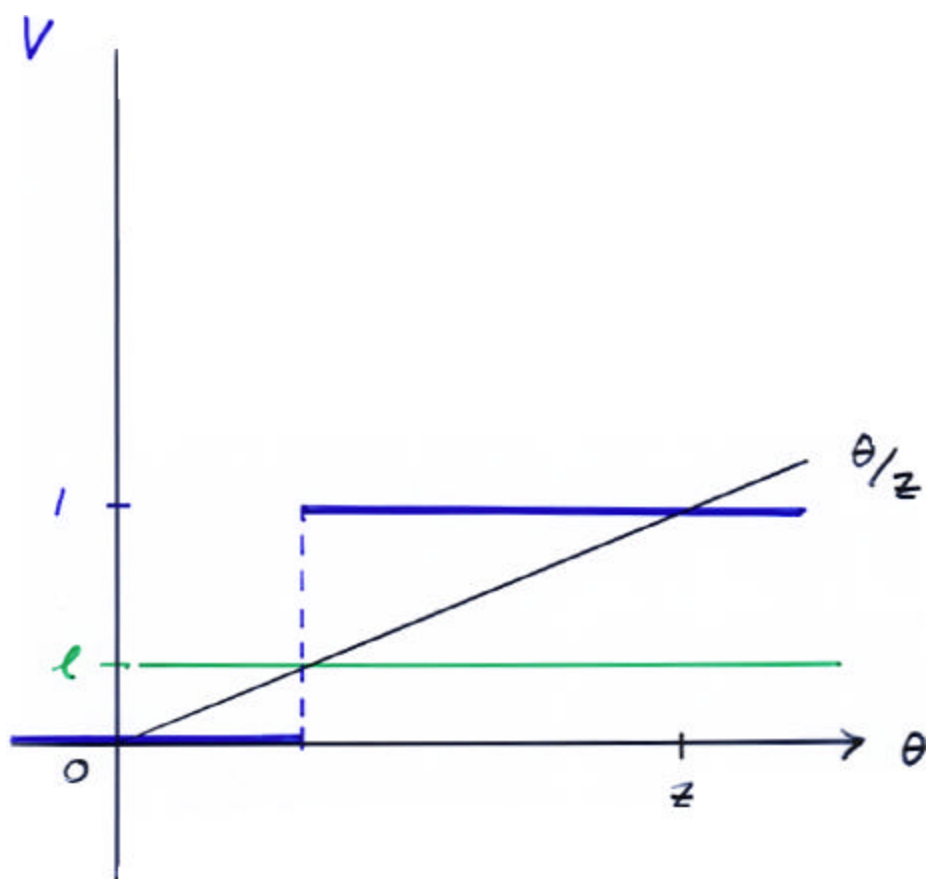


## References

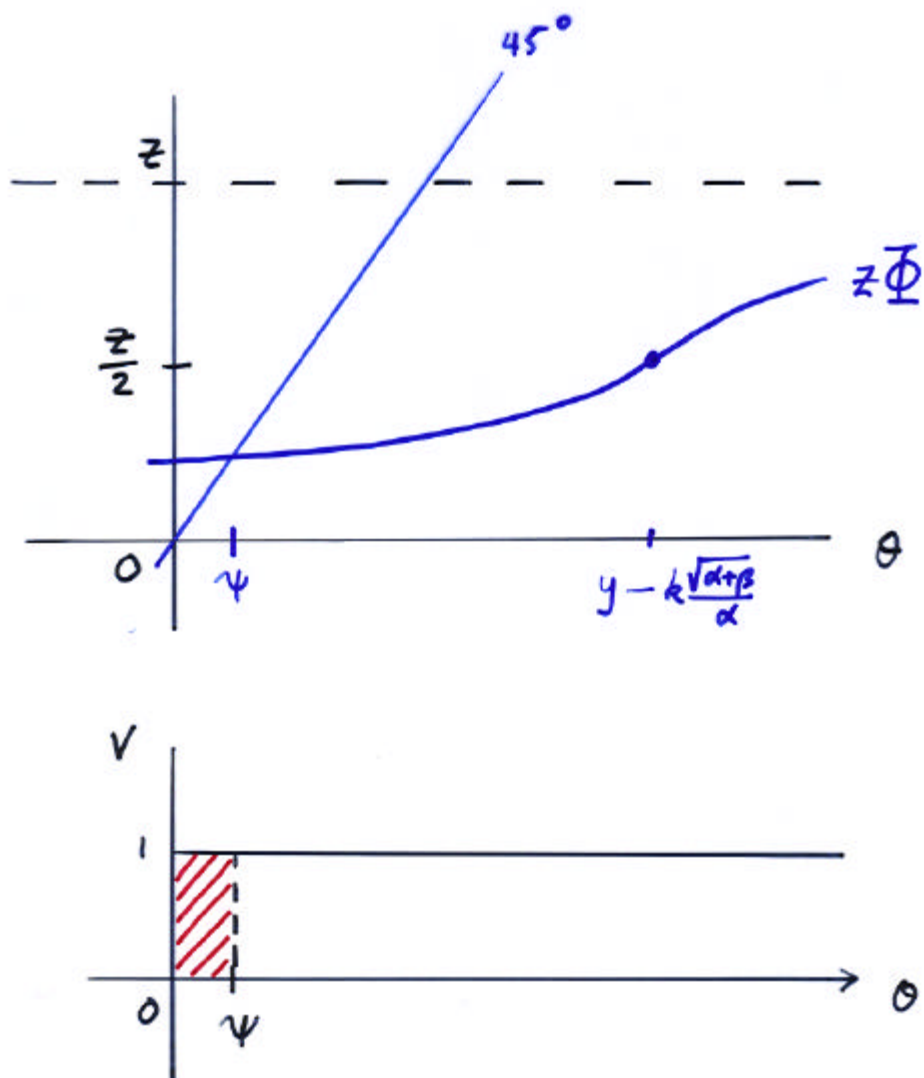
- [1] Anderson, R. and S. Sundaresan (1996) “Design and Valuation of Debt Contracts” *Review of Financial Studies*, 9, 37-68.
- [2] Baird, D. and T. Jackson (1990) *Cases, Problems and Materials on Bankruptcy*, Little Brown and Company, Boston.
- [3] Basel Committee on Banking Supervision (1999) *Recommendations for Public Disclosure of Trading and Derivatives Activities of Banks and Securities Firms*, Bank of International Settlements, Basel.
- [4] Carlsson, H. and E. van Damme (1993a). “Global games and equilibrium selection”, *Econometrica* 61, 989-1018.
- [5] Carlsson, H. and E. van Damme (1993b). “Equilibrium Selection in Stag Hunt Games,” in *Frontiers of Game Theory*, edited by K. Binmore, A. Kirman and P. Tani. MIT Press.
- [6] Diamond, D. and P. Dybvig (1983) “Bank Runs, Deposit Insurance and Liquidity” *Journal of Political Economy*, 91, 401-419.
- [7] Duffie, D. and K. Singleton (1999) “Modeling Term Structures of Defaultable Bonds” *Review of Financial Studies*, 12, 687-720.
- [8] Fischer, S. (1999) “On the Need for an International Lender of Last Resort”, paper delivered at the AEA/AFA meeting of January 1999, <http://www.imf.org/external/np/speeches/1999/010399.htm>.
- [9] Franks, J. and W. Torous (1994) “A Comparison of Financial Restructuring in Distressed Exchanges and Chapter 11 Reorganizations” *Journal of Financial Economics*, 35, 349-370.
- [10] Gertner, R. and D. Scharfstein (1991) “A Theory of Workouts and the Effects of Reorganization Law” *Journal of Finance*, 46, 1189 - 1222.
- [11] International Monetary Fund (1998a) *World Economic Outlook and International Capital Markets: Interim Assessment*, <http://www.imf.org/external/pubs/ft/weo/weo1298/index.htm>
- [12] International Monetary Fund (1998b) *Report of the Managing Director to the Interim Committee on Strengthening the Architecture of the International Monetary System*, <http://www.imf.org/external/np/omd/100198.htm>

- [13] Jackson, T. (1986) *The Logic and Limits of Bankruptcy*, Harvard University Press, Cambridge.
- [14] Jones, E., S. Mason and E. Rosenfeld (1984) "Contingent Claims Analysis of Corporate Capital Structures: an Empirical Analysis" *Journal of Finance*, 39, 611-625.
- [15] Kajii A. and S. Morris (1997) "The Robustness of Equilibria to Incomplete Information," *Econometrica*, 65 1283-1309.
- [16] Leland, H. (1994) "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure" *Journal of Finance*, 49, 1213 - 1252.
- [17] Longstaff, F. and E. Schwartz (1995) "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt" *Journal of Finance*, 50, 789 - 819.
- [18] Merton, R. C. (1974) "On the Pricing of Corporate Debt: the Risk Structure of Interest Rates" *Journal of Finance*, 29, 449-470, reprinted in *Continuous Time Finance*, Blackwell, Oxford 1990.
- [19] Milgrom, P. and J. Roberts (1990) "Rationalizability, Learning and Equilibrium in Games with Strategic Complementarities" *Econometrica*, 58, 1255 - 1278.
- [20] Monderer, D. and D. Samet (1989). "Approximating common knowledge with common beliefs", *Games and Economic Behavior* 1, 170-190.
- [21] Morris, S., R. Rob and H. S. Shin (1995), " $p$ -Dominance and Belief Potential," *Econometrica* 63, 145-157.
- [22] Morris, S. and H. S. Shin (1997) "Approximate Common Knowledge and Coordination: Recent Lessons from Game Theory," *Journal of Logic, Language and Information* 6, 171-190.
- [23] Morris, S. and H. S. Shin (1998a), "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks" *American Economic Review*, 88, 587 - 597.
- [24] Morris, S. and H. S. Shin (1998b), "A Theory of the Onset of Currency Attacks", unpublished paper, <http://www.nuff.ox.ac.uk/users/Shin/working.htm>
- [25] Nickell, P., W. Perraudin and S. Varotto (1999) "Ratings versus Equity-Based Credit Risk Modelling: an Empirical Analysis", working paper, Bank of England.

- [26] Radelet, S. and J. Sachs (1998) “The Onset of the East Asian Financial Crisis” working paper, Harvard Institute for International Development..
- [27] Rubinstein, A. (1989). “The electronic mail game: Strategic behavior under almost common knowledge,” *American Economic Review* 79, 385-391.

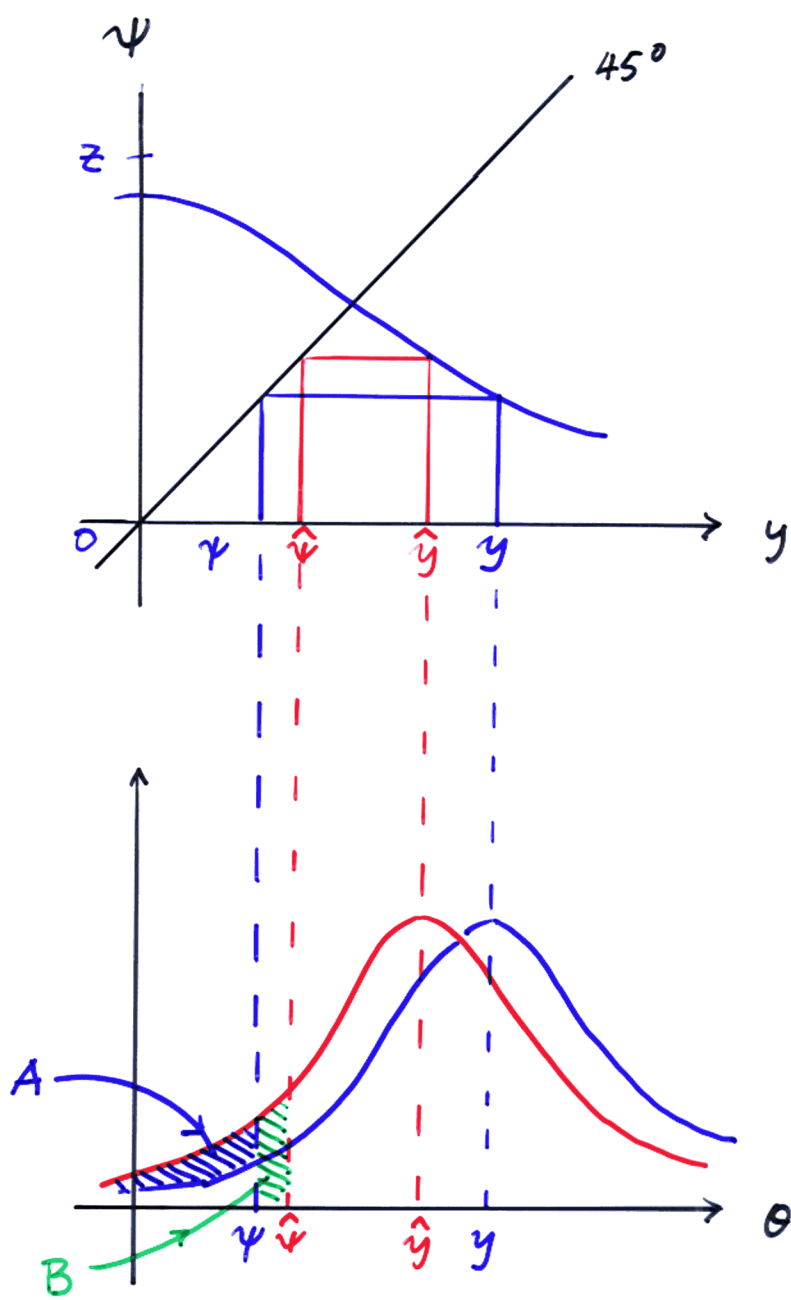


[Figure 1]



$$k = \Phi^{-1}(\lambda)$$

[Figure 2]



[Figure 4]